

PUNJAB PUBLIC SERVICE COMMISSION
COMBINED COMPETITIVE EXAMINATION FOR
RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE-2019

SUBJECT: STATISTICS (PAPER-I)

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS: 100

NOTE: Attempt Any FIVE Questions in All. Calculator is Allowed. (Not Programmable)

- Q No. 1:**
- State the properties of Arithmetic mean.
 - Calculate the variance and standard deviation from the marks obtained by 9 students
45, 32, 37, 46, 39, 36, 41, 48, 36
 - The second moment about the mean of a symmetrical distribution is 25. What must be the value of the fourth moment about the mean in order that the distribution be
 i) Leptokurtic ii) Mesokurtic iii) Platykurtic

(4 + 6 + 10 = 20 Marks)

- Q No. 2:**
- If X_1, X_2, \dots, X_n are n observations of a variable x having arithmetic mean \bar{X} , then prove that

$$\sum_{i=1}^n (x_i - \bar{x})^2 \leq \sum_{i=1}^n (x_i - a)^2$$
 - Construct a stem and leaf display for the following data.
210, 217, 208, 215, 202, 209, 207, 210, 221, 218, 212, 210, 210, 213, 200, 208, 203, 199, 218, 214.
 - Draw box and whisker plot of the following data.

Height	60-62	63-65	66-68	69-71	72-74
No. of Students	5	18	42	27	8

(05 + 05 + 10 = 20 Marks)

- Q No. 3:**
- Two dice are tossed. Let E_1 denote the event of an odd total, E_2 the event of an ace on the first die. Are E_1 and E_2 independent?
 - Given the following joint probability distribution.

$$f(x, y) = \begin{cases} (6 - x - y)/8 & \text{for } 0 \leq x \leq 2, \text{ and } 2 \leq y \leq 4 \\ 0 & \text{Elsewhere} \end{cases}$$
 Show that $f(x, y)$ is a joint probability density function. Find marginal densities $g(x)$ and $h(y)$ and conditional densities $f(x|y)$ and $f(y|x)$.

(10 + 10 = 20 Marks)

- Q No. 4:**
- The two discrete random variables x and y are such that
 $g(x) = 1/3$, for $x = 1, 2, 3$
 $h(y) = 1/2$, for $y = 0, 1$
 If $z = 2x - y$, then verify that $E(z) = 2E(x) - E(y)$

- b) Show that moment generating function of sum of two independent r.v's x and y is the product of their moment generating functions.

- c) For what value of A , the function defined below will be density function.

$$f(x) = A(x^3)(1-x) \quad 0 \leq x \leq 1$$

Find mean and variance also.

(05 + 03 + 12=20 Marks)

- Q No. 5: a) If the random variable x has binomial distribution, then show that variance is less than mean of this binomial distribution.
- b) Telephone calls being placed through a telephone exchange at random on the average of 36 calls per hour. Determine the probability,
- (i) in a 10 minutes interval, there will be 3 or more calls.
- (ii) In a 4 minutes interval, there will be no call.

(10 + 10=20 Marks)

- Q No. 6: a) Show that for normal distribution
- $$\text{Mean Deviation} = \frac{4}{5} (\text{Standard Deviation})$$
- b) In a normal distribution, $\mu = 30$ and $\sigma = 5$. Find
- (i) a point that has 15% area below it.
- (ii) probability between 20 and 40.

(10 + 10=20 Marks)

- Q No. 7: a) What is principal of least square. Use it to find normal equations of straight line $y = a + bx$
- b) The discharge of a capacitor through a resistance gave the following results. Fit a curve of the type $v = ae^{tb}$

t (Seconds)	0.5	0.8	1.4	2.0	2.5
v (Volts)	9.1	8.4	7.5	6.7	6.1

(10 + 10=20 Marks)

- Q No. 8: a) Show that $r = \sqrt{b \times d}$ for the following data.

x	2	4	5	6	8	11
y	18	12	10	8	7	5

- b) Calculate simple correlation and rank correlation for the following data.

x	4.7	2.9	6.4	2.5	4.9	7.3
y	8.6	5.4	6.2	4.9	8.3	7.2

(08 + 12=20 Marks)

PUNJAB PUBLIC SERVICE COMMISSION
COMBINED COMPETITIVE EXAMINATION FOR
RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE-2019

SUBJECT: STATISTICS (PAPER-II)

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS: 100

NOTE: Attempt Any FIVE Questions in All. Calculator is Allowed. (Not Programmable)

- Q No. 1:**
- Define sampling and non-sampling errors and how we can control these errors?
 - A random sample of size $n = 100$ is taken from a population having a mean of 20 and a standard deviation of 5. The shape of the population distribution is unknown.
 - What can you say about the sampling distribution of the sample mean \bar{X} ?
 - Find the probability that \bar{X} will exceed 20.75.
 - A population consists $N = 6$ elements 0, 3, 4, 6, 9 and 15. Draw all possible samples of size $n = 3$, without replacement, from the population and find the sample proportion of even numbers in the samples. Construct the sampling distribution of sample proportions and verify that

$$E(\hat{p}) = p \text{ and } Var(\hat{p}) = \frac{pq}{n} \cdot \frac{N-n}{N-1}$$

(5+5+10=20 Marks)

- Q No. 2:**
- Explain the desirable properties of a good point estimator.
 - In order to ascertain the age distribution of operatives in a certain industry, random samples of 1720 males and 1230 females are drawn. The sample means and standard deviations were 33.93 years and 14.20 years for the males, and 27.44 years and 10.79 years for the females. Calculate the 95% confidence interval estimates for
 - the mean age of all male operatives,
 - the differences between their mean ages.
 - In a random sample of 1000 homes in a certain city, it is found that 628 have the phone connections. Find 95% confidence interval for the proportion of homes in this city that have phone connections.

(5+10+5=20 Marks)

- Q No. 3:**
- The manufacturer of a patent medicine claimed that it was 90% effective in relieving an allergy for a period of 8 hours. In a sample of 200 people who had the allergy, the medicine provided relief for 160 people. Determine whether the manufacturer's claim is legitimate at the $\alpha = 0.01$ level.
 - The weights of 4 persons before they stopped smoking and 5 weeks after they stopped smoking are as follows:

Person	1	2	3	4
Before	148	176	153	116
After	154	176	151	121

Use the t -test for paired observations to test the hypothesis at the 0.05 level of significance, that giving up smoking has no effect on a person's weight

(10+10=20 Marks)

- Q No. 4:**
- What are the assumptions underlying a one-way analysis of variance?
 - Twenty men are used in an experiment, five being assigned at random to each of the four machines. The observations are the amounts produced by the machines in one day. Test the hypothesis at $\alpha = 0.05$, that the machines are not different with respect to the number of items produced.

Machine Number			
1	2	3	4
64	41	65	45
39	48	57	51
65	41	76	55
46	49	72	48
63	57	64	47

(5+15=20 Marks)

- Q No. 5:** a) Describe a Latin Square design and its analysis. What are the advantages and disadvantages of a Latin Square design?
 b) Carry out the analysis of variance for the following Latin Square:- (10+10=20 Marks)

V_1 (2.3)	V_2 (3.0)	V_3 (3.3)	V_4 (2.5)
V_2 (3.1)	V_3 (4.1)	V_4 (2.4)	V_1 (2.4)
V_3 (4.3)	V_4 (2.5)	V_1 (2.1)	V_2 (2.9)
V_4 (2.6)	V_1 (2.0)	V_2 (2.4)	V_3 (4.4)

- Q No. 6:** a) Compute the consumer price index number for the following data for 2018 with 2017 as base. Use as weights (i) the quantities consumed in the base year, (ii) the value in the base year,

Article	Quantity (units) 2017	Price (in dollars)	
		2017	2018
A	5	18.00	26.50
B	30	2.60	2.80
C	75	0.25	0.30
D	3	30.00	27.50
E	34	0.50	0.60

- b) The following table gives the numbers of deep freezers sold by a certain company:

Year	Quarters			
	I	II	III	IV
2016	42	45	33	36
2017	30	45	40	65
2018	51	69	65	94

Use the trend equation and the seasonal index to forecast the sales for each quarter of the year 2019. (8+12=20 Marks)

- Q No. 7:** a) What are the various statistical organizations in Pakistan?. Describe the function of any one of these main organizations?
 b) Calculate age-specific fertility rates, total fertility rate, gross reproduction rate and net reproduction rate for the following data. assume sex ratio at birth to be 105.2 per cent.

Age-group (years)	Female Population	Registered Births	Probability of Survival
15-19	13,472	1,223	0.9694
20-24	8,812	1,744	0.9668
25-29	5,419	1,709	0.9632
30-34	5,070	708	0.9584
35-39	5,018	369	0.9519
40-44	4,860	25	0.9424

(8+12=20 Marks)

- Q No. 8:** a) The following table gives the census data of orchards. Test the hypothesis that the two variables of classification are independent.

Classes	Shaded	Unshaded	Total
High Yielders	350	205	555
Low Yielders	250	195	445
Total	600	400	1000

- b) Given below are the grade point averages received by two groups of students.

Group 1	3.1, 5.3, 6.4, 6.2, 3.8, 7.5, 5.8, 4.3, 5.9, 4.9
Group 2	9.0, 5.6, 6.3, 8.5, 4.6, 7.1, 5.5, 7.9, 6.8, 5.5, 8.9

Test by the Mann-Whitney U test the hypothesis that the two groups come from identical populations. Use a 0.05 significance level.

(10+10=20 Marks)