PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION FOR RECRUITMENT TO THE POSTS OF PROVINCIAL MANAGEMENT SERVICE -2020

SUBJECT: MATHEMATICS (PAPER-I)

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE:

i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
iv. Extra attempt of any question or any part of the question will not be considered.

ATTEMPT FIVE QUESTIONS IN ALL. THREE QUESTIONS FROM SECTION "A" AND TWO QUESTIONS FROM SECTION "B". CALCULATOR IS ALLOWED (NOT PROGRAMMABLE).

SECTION-A

Q 1: (a) Evaluate the limit

$$\lim_{y\to x}\frac{y^{2/3}-x^{2/3}}{y-x}$$

(b) If $= (\sin x)^{\cos x} + (\cos x)^{\sin x}$, then find $\frac{dy}{dx}$ where $\sin x$ and $\cos x$ are non-negative. **(10 + 10 = 20 Marks)**

Q 2: (a) Show that

$$\int_0^{\pi/2}\frac{sin^2x}{\sin x+\cos x}\,dx=\frac{1}{\sqrt{2}}\,ln(\sqrt{2}+1)$$

(b) Use the Mean Value Theorem to show that $|\tan x + \tan y| \geq |x+y|$ for all real numbers x and y in the interval $\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$ **(10+10=20 Marks)**

Q 3: (a) Find the relative extreme values of

$f(x) = 12x^5 - 45x^4 + 40x^3 + 6$

(b) With $n = 8$, find the area under the semi-circle $y=\sqrt{4-x^2}$ and above the x-axis by Simpson's Rule. **(10+10=20 Marks)**

Q 4: (a) Solve the differential equation $x\frac{dy}{dx}-(1+x)\,y=x\,y^2$

(b) Solve initial value problem

$4y''+4y'+17y=0, \quad y(0)=-1, \quad y'(0)=2$ **(10+10=20 Marks)**

Q 5: (a) Solve the differential equation by the method of Undetermined Coefficients

$$y''-3y'=8e^{3x}+4\sin x$$

(b) A mass weighing 8 pounds stretches a spring 2 feet. Assuming that a damping force numerically equal to 2 times the instantaneous velocity acts on the system, determine the equation of the motion if the mass is initially released from the equilibrium position with an upward velocity of 3ft/s.

(10+10 = 20 Marks)

SECTION-B

(a) Determine whether the following series are convergent or divergent.

(5+5=10 Marks)

(i) $\sum_{n=1}^{\infty}\frac{(n+3)!}{3!\cdot n!\cdot 3^n}$ (ii) $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$

(b) Find radius and Interval of convergence of the series **(10 Marks)**

$$\sum_{n=0}^{\infty}\frac{n\cdot(x+3)^n}{5^n}$$

Q 7: (a) Determine whether the given function is Analytic or not at any point

$$f(z)=e^{-y}\sin x - i\,e^{-y}\cos x$$

(b) Evaluate the integral $\int_c \frac{dz}{z^2+2z+2}$

where c is a square having corners as $(0,0),(0,-2),(-2,0),(-2,-2)$.

(10 + 10 = 20 Marks)

Q 8:- (a) Determine the focus, vertex and directrix of the parabola

$$y^2-2y+16x=-49$$

Also sketch its graph.

(b) Find the Tangent (T), Normal (N), Binormal(B) , Curvature(κ)

$r(t)=(e^t\cos t)\,i+(e^t\sin t)\,j+2\,k$

(10 + 10 = 20 Marks)

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COMBINED COMPETITIVE EXAMINATION
FOR RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE -2020

SUBJECT: MATHEMATICS (PAPER-II)

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE:
i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
iv. Extra attempt of any question or any part of the question will not be considered.

Attempt any five questions. All questions carry equal marks. Calculator is allowed (Not Programmable).

Q1:(a) Determine whether the following vectors in R^4 are linearly independent or dependent $(1,-2,4,1),\ (2,1,0,-3), (1,-6,1,4)$.

(b) If U, W are finite dimensional subspaces of a vector space V over a field F, then $\dim(U+W) = dimU + dimW - \dim(U \cap W)$. **(10+10=20)**

Q2:(a) Show that the vectors $(3+\sqrt{2},\ 1+\sqrt{2})$ and $(7,\ 1+2\sqrt{2})$ in R^2 are linearly dependent over R but linearly independent over Q.

(b) Let $R = \{a + b\sqrt{5} \mid a, b \in Q\}$. Then show that $(R,\ +,\ \bullet)$ is a ring, where + and $\bullet$ denote ordinary addition and ordinary multiplication respectively. **(10+10=20)**

Q3:(a) Let G be a group such that $(ab)^n = a^n b^n$ for three consecutive numbers and for all $a, b \in G$. Show that G is an abelian group.

(b) Show that
$G = \{2^k \mid k = 0, \pm1, \pm2, \dots\dots\dots\}$ is a group under the ordinary multiplication. **(10+10=20)**

Q4:(a) The union $H \cup K$ of two subgroups H and K of a group G is a subgroup of G if and only if, either $H \subset K$ or $K \subset H$.

(b) Prove that if every non-identity element of a group G is of order 2, then G is abelian. **(10+10=20)**

Q5:(a) Find the Eigen values and corresponding Eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(b) For what values of λ does the following homogeneous system of equations have non trivial solution? Find these solutions.

$$(3-\lambda)x - y + z = 0$$
$$x - (1-\lambda)y + z = 0$$
$$x - y + (1-\lambda)z = 0.$$

(10+10=20)

P.T.O

Q 6:(a) If A and B are 3×3 matrices such that $\det(A^2B^2) = 108$ and $\det(A^3B^2) = 72$, find $det(2A)$& $\det(B^{-1})$.

(b) Write matrix (A) given below as the sum of symmetric and skew symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}.$$

(10+10=20)

Q7:(a) Prove that co-finite topology is discrete if and only if X is finite.

(b) Show that $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$ **(10+10=20)**

Q 8:(a) Let X denote the set of all continuous real valued functions defined on $[a,b]$. Define $d: X \times X \to R$ by

$$d(x,y) = \int_a^b |x(t) - y(t)|dt.$$

To show d is a metric on X.

(b) Let $X = \{a,b,c,d,e\}$, $\tau = \{\phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,e\}, \{a,b,c,d\}, X\}$ be a topology on X. Find the neighborhood system of each of its points.

(10+10=20)