PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION FOR RECRUITMENT TO THE POSTS OF

PROVINCIAL MANAGEMENT SERVICE-2019

SUBJECT: MATHEMATICS (PAPER-I)

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt FIVE Questions in All. THREE Questions from Section 'A' and TWO Questions from Section 'B'. Calculator is allowed. (not programmable)

SECTION-A

Q 1:- (a) Evaluate the following limit

$$\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}$$

(b) Find $\frac{dy}{dx}$ for the given $y = x^x e^x \sin(\ln x)$ **(10+10=20 Marks)**

Q 2:- (a) Evaluate $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$

(b) Solve $x\frac{dy}{dx}+y=x^2y^2$ **(10+10=20 Marks)**

Q 3:- (a) Find the solution of the following initial value problem

$$y'''+12y''+36y'=0\ ;\ \ y(0)=0,\ y'(0)=1,\ y''(0)=-7$$

(b) Solve the given differential equation by the method of undetermined coefficients

$$y^{(4)}+y'''=1-x^2e^{-x}$$

(10+10=20 Marks)

Q 4:- (a) A mass weighing 24 pounds, attached to the end of a spring, stretches it 4 inches. Initially, the mass is released from rest from a point 3 inches above the equilibrium position. Find the equation of motion.

(b) Find equations of the asymptotes of the following curve

$$y(x-y)^2 = x+y$$

(10+10=20 Marks)

Q 5:- (a) The velocity of a car travelling on a motorway at 15 minute intervals is as follows

Time in Hours (t)	¼	½	¾	1	5/4
Velocity in (Km)	100	90	115	120	80

Using Trapezoidal rule, find total distance travelled by the car in 60 minutes period from $t=\frac{1}{4}$ to $t=\frac{5}{4}$.

(b) Find c of the mean value theorem for $f(x)=x^3-3x-1$ on [-11/7, 13/7]

(10+10=20 Marks)

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SECTION-B

Q 6:- (a) Investigate the convergence of the series $\sum_{n=0}^{\infty}\frac{2^n+5}{3^n}$.

(b) Find the Taylor series generated by $f(x)=e^x$ at $x=0$.

(10+10=20 Marks)

Q 7:- (a) Show that the given function is not analytic at any point

$f(z)=\frac{x}{x^2+y^2}+i\frac{y}{x^2+y^2}$

(b) Evaluate $\oint_c \frac{z+1}{z^4+2iz^3}dz$ where c is the circle $|z|=1$.

(10+10=20 Marks)

Q 8:- (a) Examine whether the following equation represents two straight lines. If so, find the equations of each straight line $6x^2-17xy-3y^2+22x+10y-8=0$

(b) Prove that the radius of curvature at the point $(2a,2a)$ on the curve $x^2y=a(x^2+y^2)$ is $2a$.

(10+10=20 Marks)

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COMBINED COMPETITIVE EXAMINATION FOR RECRUITMENT TO THE POSTS OF PROVINCIAL MANAGEMENT SERVICE-2019

SUBJECT: MATHEMATICS (PAPER-II)

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt FIVE Questions in All. Calculator is allowed (not programmable)

Q 1:- a) Let G be a cyclic group of order n generated by a .Then, for each positive divisor d of n, there is a unique subgroup ($of\ G$) of order d .

b) Prove that both the order and index of a subgroup of a finite group divide the order of the group. **(10+10=20 Marks)**

Q.2:- a) Let H , K be two subgroups of a finite group G . Prove that for any $g \in G$
$g(H \cap K) = gH \cap gK$

b) Prove that the set S_n of all permutations on a set X with n elements is a group under the operation of **composition** of permutations.

(10+10=20 Marks)

Q.3:- a) If G is an abelian group, show that $(ab)^n = a^n b^n$ for all $a, b \in G$

b) If U , W are sub spaces of a vector space V , then $U + W$ is a sub space of V containing both U and W. Further, $U + W$ is the smallest sub space containing both U and W . **(10+10=20 Marks)**

Q.4:- a) Find an equation (or equations) of the subspace W of R^3 spanned by the following set of
Vectors $\{(1,-2,1),(-2,0,3),(3,-2,-2)\}$

b) Let $T: R^3 \to R^3$ be the linear transformation defined by

$$T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3).$$

Find a basis and dimension of $R(T)$ and $N(T)$. **(10+10=20 Marks)**

Q.5:- a) Prove that every closed ball in a metric space (X, d) is closed.

b) Let (X, d) be a metric space. Define $d': X \times X \to \mathbb{R}$ by:

$$d'(x, y) = \min(d(x, y), 1)$$

Show that (X, d') is a metric space. **(10+10=20 Marks)**

Q.6:- a) Find a real orthogonal matrix P for which $P^{-1} A P$ is diagonal where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

b) Find the Eigen-values and corresponding Eigen-vectors of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

(10+10=20 Marks)

P.T.O

Q.7:- a) Reduce the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ to the normal form. Also find the non-singular matrices P and Q such that PAQ is in the normal form.

b) For what value of λ the equations

$$(5-\lambda)x_1 + 4x_2 + 2x_3 = 0$$

$$4x_1 + (5-\lambda)x_2 + 2x_3 = 0$$

$$2x_1 + 2x_2 + (2-\lambda)x_3 = 0$$

have nontrivial solutions. Find these solutions.

(10+10=20 Marks)

Q.8:- a) Solve by Gauss-Seidel method, the following systems of equations.

$$28x + 4y - z = 32$$

$$x + 3y + 10z = 24$$

$$2x + 17y + 4z = 35$$

b) Prove by using properties of determinants

$$\begin{vmatrix} a^3 & 3a^2 & 3a & 1 \\ a^2 & a^2+2a & 2a+1 & 1 \\ a & 2a+1 & a+2 & 1 \\ 1 & 3 & 3 & 1 \end{vmatrix} = (a-1)^6$$

(10+10=20 Marks)