FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2025 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT
PURE MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. No.1.(a) Let Q^+ be the set of positive rational numbers and define $*$ by $a * b = \frac{ab}{2}$ then prove that $(Q^+, *)$ is a group. **(10)**

(b) Find all the cyclic subgroups of Z_{18}. **(10)**

Q. No.2. (a) A homomorphism $\varphi: Z_6 \to Z_6$ is a one to one mapping then calculate Ker(φ). **(10)**

(b) Check whether the vectors $a = (1, 2, 3), b = (2, 5, 7) and\ c = (1, 3, 5)$ are linearly dependent or independent. **(10)**

Q. No.3. (a) Let V be a vector space of all 2×2 matrices. W be a subspace of V which consists of all symmetric matrices then find two different basis of W. **(10)**

(b) Consider the transformation $T: R^3 \to R^2$ given by $T(x, y, z) = (|x|, y+z)$. Check whether T is linear or not. **(10)**

SECTION-B

Q. No.4. (a) Find all the real numbers $x \in R$ such that $x^2 + x > 2$. **(10)**

(b) Use the definition of limit to establish that $\lim_{x\to 2} \frac{x^3-4}{x^2+1} = \frac{4}{5}$. **(10)**

Q. No.5. (a) Use Mean Value Theorem to show that $e^x \geq 1 + x\ \forall\ x \varepsilon R$. **(10)**

(b) Find the absolute extrema of the function $f(x, y) = xy - 2x$ on the region R given by vertices (0, 4), (4, 0) and (0, 0). **(10)**

Q. No.6. (a) Change the order of integration in double integral $\int_0^2 \int_0^{\sqrt{x}} f(x, y)\,dydx$. **(10)**

(b) Draw the graph of the conic $r = 2Cos\theta$. **(10)**

SECTION-C

Q. No.7. (a) Check that the Cauchy-Riemann Equations are satisfied in polar coordinates for $f(Z) = \frac{1}{Z}$. **(10)**

(b) Evaluate the contour integral $\oint_C f(Z)dZ$ where $f(Z) = y - x - 3ix^2$ and C is simple closed contour OABO with O = 0+0i, A = 0+i and B = 1+i. **(10)**

Q. No.8. (a) Use Cauchy Residue Theorem to evaluate the integral $\int_C \frac{5Z-2}{Z(Z-1)}dZ$ where C is the circle $|Z| = 2$. **(10)**

(b) Find the Maclaurin series for the function $f(Z) = Z^2e^{3Z}$. **(10)**

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