**FEDERAL PUBLIC SERVICE COMMISSION**
**COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT**
**TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT**

**Roll Number**

# PURE MATHEMATICS

| **TIME ALLOWED: THREE HOURS** | **MAXIMUM MARKS = 100** |
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**NOTE:**
- **(i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
- **(ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.
- **(iii)** Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- **(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- **(v)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(vi)** **Use of Calculator is allowed.**

## SECTION-A

**Q. 1.** **(a)** Find centre of $S_3$. (10)

**(b)** Using the row operations, show that the matrix $\begin{pmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{pmatrix}$ has no inverse. (10) **(20)**

**Q. 2.** **(a)** For any group $G$, show that $\frac{G}{\{e\}} \cong G$ and $\frac{G}{G} \cong \{e\}$. (10)

**(b)** Suppose $U$ and $W$ are distinct four dimensional subspaces of a vector space $V$ of dimension six. Find the possible dimension of $U \cap W$. (10) **(20)**

**Q. 3.** **(a)** For what value of $\alpha$ is the matrix $\begin{pmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{pmatrix}$ is singular? (10)

**(b)** Define $T: \Re^3 \to \Re^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is $T$ one-to-one? (10) **(20)**

## SECTION-B

**Q. 4.** **(a)** Find the value of $\theta$ and the limit in order that $\lim\limits_{x \to 0} \frac{\sin 2x + \theta \sin x}{x^3}$ be finite. (10)

**(b)** Show that $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$, 0<x<1. (10) **(20)**

**Q. 5.** **(a)** Given that $U = \frac{1}{\sqrt{x^2+y^2+z^2}}$. Verify that $U_{xx} + U_{yy} + U_{zz} = 0$. (10)

**(b)** Evaluate $\iint (x^2+y^2)\,dxdy$, over the domain bounded by $y = x^2$ and $x = y^2$. (10) **(20)**

**Q. 6.** **(a)** Evaluate $\iint (x^2+y^2)\,dxdy$, over the region bounded by xy=1, y=0, y=x and x=2. (10)

**(b)** Find an equation of a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in the form $ax\cos\theta + by\cot\theta = a^2 + b^2$. Prove that the normal is external bisector of the angle between the focal distances of its foot. (10) **(20)**

## SECTION-C

**Q. 7.** **(a)** Determine k such that $U = e^{2x}\cos ky$ is harmonic and find a conjugate harmonic. (10)

**(b)** Evaluate $\int_C (\frac{1}{z^5} + z^3)dz$ from 1 to -1 along the upper arc of the unit circle. (10) **(20)**

**Q. 8.** **(a)** Find the Laurent Series of $\frac{1}{1-z^2}$ in the region $0<|z-1|<2$. (10)

**(b)** Find the residues at the singular points of $\frac{-Z^2-22z+8}{Z^3-5z^2+4z}$ which lie inside the circle $|z|=2$. (10) **(20)**

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