FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2022 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) Use of Calculator is allowed.

SECTION-A

Q. 1. (a) Let G be a group and H be a subgroup of index 2 in G. Show that H is normal in G. (10)

(b) Let G be any group, g a fixed element in G. Define $\phi: G \to G$ by $\phi(x) = gxg^{-1}, \forall x \in G$. Prove that ϕ is an automorphism of G onto G. (10) **(20)**

Q. 2. (a) Prove that a finite integral domain is a field. (10)

(b) Let W be the subspace of R^5 spanned by $u_1 = (1,2,-1,3,4), u_2 = (2,4,-2,6,8)$, $u_3 = (1,3,2,2,6), u_4 = (1,4,5,1,8), u_5 = (2,7,3,3,9)$. Find a subset of the vectors that form a basis of W. Also extend the basis of W to a basis of R^5. (10) **(20)**

Q. 3. (a) Let $T: R^4 \to R^3$ be defined by (10)

$$T(x,y,z,t) = (x - y + z + t,\ 2x - 2y + 3z + 4t,\ 3x - 3y + 4z + 5t)$$

Find the rank and nullity of T.

(b) Find all possible solutions of the following homogeneous system of equations. (10) **(20)**

$$x_1 + x_2 + x_3 - x_4 = 0$$
$$x_1 + 2x_2 - 2x_3 + x_4 = 0$$
$$2x_1 + 4x_2 - 3x_3 + x_4 = 0$$
$$4x_1 + 7x_2 - 4x_3 + x_4 = 0$$

SECTION-B

Q. 4. (a) Find $\lim_{x\to\infty} (1 + 2x)^{1/(2\ln x)}$. (10)

(b) Evaluate the integral $\int e^{3x} \cos 2x\, dx$. (10) **(20)**

Q. 5. (a) If $u = f(x,y)$ and $x = r\cos\theta, y = r\sin\theta$, then show that (10)

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) Evaluate $\iint_R x\, dx\, dy$ over the region bounded by $y = x^2$ and $y = x^3$. (10) **(20)**

Q. 6. (a) Find the area of the region bounded above by $y = x + 6$, bounded below by $y = x^2$, and bounded on the sides by the lines $x = 0$ and $x = 2$. (10)

(b) Find the foci, vertices and center of the ellipse: (10) **(20)**

$$9x^2 + 16y^2 - 72x - 96y + 144 = 0$$

SECTION-C

Q. 7. **(a)** Prove that the function $u(x,y) = e^{-x}(x\sin y - y\cos y)$ is harmonic. Also find a function $v(x,y)$ such that $f(z) = u(x,y) + i\,v(x,y)$ is analytic. (10)

(b) Evaluate $\oint_C \bar{z}^2 dz$ around the circle $|z| = 1$. (10) **(20)**

Q. 8. **(a)** Use residues to prove that (10)

$$\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$$

(b) Find the Fourier series of the following function $f(x)$ which is assumed to have the period 2π. $f(x) = |x|, -\pi < x < \pi$ (10) **(20)**

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