



FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2019
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT
PURE MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100
<p>NOTE: (i) Attempt FIVE questions in all by selecting TWO Questions each from SECTION-A&B and ONE Question from SECTION-C. ALL questions carry EQUAL marks.</p> <p>(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.</p> <p>(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.</p> <p>(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.</p> <p>(v) Extra attempt of any question or any part of the attempted question will not be considered.</p> <p>(vi) Use of Calculator is allowed.</p>	

SECTION-A

- Q. 1.** (a) Show that the order and the index of a subgroup divides the order of a finite group. (10)
- (b) Show that every finite integral domain is a field. (10) (20)
- Q. 2.** (a) Show that the characteristic of an integral domain is R is either zero or a prime. (10)
- (b) Determine whether or not the set $\{(1, 2, -1), (0, 3, 1), (1, -5, 3)\}$ of vectors is a basis for R^3 . (10) (20)
- Q. 3.** (a) Show that a one-to-one linear transformation preserves basis and dimension. (10)
- (b) Solve the system of linear equations: (10) (20)
- $$\begin{aligned} 2x_1 + x_2 + 5x_3 &= 4 \\ 3x_1 - 2x_2 + 2x_3 &= 2 \\ 5x_1 - 8x_2 + 2x_3 &= 1. \end{aligned}$$

SECTION-B

- Q. 4.** (a) Solve $\int_0^{\frac{\pi}{2}} \sin^2 6x \cos^4 3x dx$. (10)
- (b) Find the area enclosed by $y = \frac{6}{2 - \cos \theta}$. (10) (20)
- Q. 5.** (a) Show that in any conic semi-latusrectum is the harmonic mean between the segments of focal chord. (10)
- (b) Prove that the evolute of hyperbola (10) (20)
- $$2xy = a \text{ is } (x + y)^{\frac{2}{3}} - (x - y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}.$$
- Q. 6.** (a) Define Supremum and Infimum of a sequence. Find the supremum and infimum of the set (10)
- $$\left\{ (-1)^n \left(1 - \frac{1}{n} \right), n = 1, 2, 3, \dots \right\}.$$
- (b) Evaluate (10) (20)
- $$\lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}.$$

SECTION-C

- Q. 7.** (a) Show that $\text{Log}(1 + \cos \theta + i \sin \theta) = \ln(2 \cos \frac{\theta}{2}) + i \frac{\theta}{2}$. (10)
- (b) Find v such that $f(z) = u + iv$ is analytic. (10) **(20)**
- Q. 8.** (a) Prove that the series $z(1 - z) + z^2(1 - z) + z^3(1 - z) + \dots$ converges for $|z| < 1$, and find its sum. (10)
- (b) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. (10) **(20)**
