FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION-2025 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt only **FIVE** questions in all. **ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

Q. No. 1 **(a)** **(i)** Prove that $\nabla r^n = nr^{n-2}\vec{r}$, *where* $\vec{r} = x\underline{i} + y\underline{j} + z\underline{k}$. **(10)**

(ii) $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \left(\vec{a}\times\vec{b}\right)\times\vec{c}$, then prove that $\vec{a}$ and $\vec{c}$ are parallel.

(b) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x+6$. **(10)**

Q. No. 2 **(a)** Find the tangential and normal components of acceleration of a point describing the ellipse **(10)**

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ with uniform speed } \vec{V}\text{, when the particle is at } (0,\ b).$$

(b) Find the solution of initial value problem by separation of variables **(10)**

$$\sqrt{1-y^2}\,dx - \sqrt{1-x^2}\,dy = 0, \qquad y(0)=\frac{\sqrt{3}}{2}$$

Q. No. 3 **(a)** Find the general solution of the given differential equation by variation of parameters. **(10)**

$$3y'' - 6y' + 6y = e^x \sec x$$

(b) Find the power series solution of $\left(x^2+1\right)y'' + xy' - y = 0$ **(10)**

Q. No. 4 **(a)** Forces $2\overrightarrow{BC},\ \overrightarrow{CA},\ \overrightarrow{BA}$ act along the sides of a triangle ABC. Show that their resultant is $6\overrightarrow{DE}$. Where D bisects BC and E is a point on CE such that $CE = \frac{1}{3}CA$. **(10)**

(b) Find the center of mass of the surface generated by the revolution of the arc of the parabola, lying between the vertex and the latus rectum, about the x-axis. **(10)**

Q. No. 5 **(a)** Obtain the Fourier series over the indicated interval for the given function. **(10)**

$$\begin{aligned} f(x) &= 3\pi + 2x, && -\pi < x < 0, \\ &= \pi + 2x, && 0 < x < \pi \end{aligned}$$

(b) Solve the boundary value problem **(10)**

$$\begin{aligned} u_{xx} + u_{yy} &= 0, && 0 < x < a, \quad 0 < y < b, \\ u(0,y) &= 0, && u(a,y) = 0, \quad 0 \le y \le b \\ u(x,0) &= 0, && u(x,b) = f(x), \quad 0 \le x \le a. \end{aligned}$$

Q. No. 6 **(a)** Use Newton's Raphson method to find the solution accurate to within 10^{-4} (corrected upto four decimal places) for the given problem. **(10)**

$$x - \cos x = 0, \qquad [0,\ \pi/2].$$

(b) Solve the system of linear equations using Gauss Seidel method (with three digit rounding arithmetic) **(10)**

$$\begin{aligned} 3x_1 + 4x_2 - x_3 &= 8 \\ 5x_1 + 3x_2 + 2x_3 &= 17 \\ -x_1 + x_2 - 3x_3 &= -8 \end{aligned}$$

Q. No. 7 **(a)** Use Euler's method to approximate the solution of the initial value problem. **(10)**

$$y' = 1 + y/x, \quad 1 \le x \le 2, \; y(1) = 2, \; with \; h = 0.25$$

(b) Using Green's theorem, evaluate $\int_C \vec{F}(\vec{r}).\, d\vec{r}$ counter clock wise around the boundary curve C of the region R, where $\vec{F} = \left[\frac{1}{2}xy^4, \; \frac{1}{2}x^4 y\right]$, R the rectangle with vertices $(0,\,0),\,(3,\,0),\,(3,\,2),\,(0,\,2)$. **(10)**

Q. No. 8 **(a)** Evaluate the Integral $\int_1^3 \frac{1}{x^2}\,dx$, Using Trapezoidal Rule for five points (corrected upto two decimal places). **(10)**

(b) Find the D'Alembert solution of the wave equation $u_{xx} = \frac{1}{c^2}u_{tt}$ subject to the Cauchy Initial conditions $u(x,\,0) = f(x), \;\; u_t(x,\,0) = g(x)$. **(10)**

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