



FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT
TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

- NOTE:** (i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

- Q. No. 1.** (a) Forces of magnitudes P , $2P$, $3P$, $4P$ act respectively along the sides AB , BC , CD , DA of a square $ABCD$, of side a and forces each of magnitude $(8\sqrt{2})P$ act along the diagonals BD , AC . Find the magnitude of the resultant force and the distance of its line of action from A . (10)
- (b) A uniform rod AB of length a and weight W is freely hinged to a vertical wall at A and is maintained in equilibrium by a light string of length a fastened to B and to a point C at a distance b vertically above A . Prove that the reaction at the hinge A is (10)

$$W \frac{\sqrt{(a^2 + 2b^2)}}{2b}$$

and find the tension in the string.

- Q. No. 2.** (a) Use Runge-Kutta method of order two to solve the following differential equation at $x=1.2$ by taking $h=0.1$ (10)
- $$\frac{dy}{dx} = \frac{3x+y}{x+2y} \quad y(1) = 1.$$
- (b) Find the first and second derivatives of $f(x)$ at $x = 3$ from the following data using Newton's forward difference interpolation formula (10)

x	3	3.5	4	4.5	5	5.5
$f(x)$	4.1023	5.1047	8.1971	9.1096	4.1122	6.1148

- Q. No. 3.** (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (8)
- (b) Show that (6)
- $$\nabla r^n = nr^{n-2}r$$
- (c) Find the total work done in a moving particle in a force field given by (6)
- $$F = 3xy \mathbf{i} - 5z \mathbf{j} + 10x \mathbf{k}$$
- along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

- Q. No.4.** (a) A particle P moves in a plane in such a way that at any time t , its distance from a fixed point O is $r = at + bt^2$ and the line connecting O and P makes an angle $\theta = ct^{\frac{3}{2}}$ with a fixed line OA . Find the radial and transverse components of the velocity and acceleration of the particle at $t = 1$. (10)
- (b) Solve the following Bernoulli's equation (10)
- $$x \frac{dy}{dx} + y = \frac{1}{y^2}$$

- Q. No. 5.** (a) Solve the following differential equation (10)
- $$x dy = (x \sin x - y) dx$$
- (b) Find the general solution of the higher order differential equation (10)
- $$y''' + 8y'' = -6x^2 + 9x + 2$$

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Q. No. 6. (a) Find solution of $4y'' + y = 0$ in the form of power series in x . **(10)**

(b) Solve the following differential equation by variation of parameters **(10)**

$$y'' - 4y' + 4y = (x + 1)e^{2x}$$

Q. No. 7. (a) Find real root of the equation $2x - 3 \sin(x) - 5 = 0$ up to 4 decimal places by secant method. **(10)**

(b) Solve the following system of equations by Gauss Seidel method. Perform only five iterations. **(10)**

$$8x_1 - x_2 - x_3 = 6$$

$$x_1 + 6x_2 + x_3 = 8$$

$$x_1 - x_2 + 5x_3 = 5$$

Q. No. 8. (a) Expand $f(x) = \sin x$, $0 < x < \pi$, in a Fourier cosine series. **(10)**

(b) Use the method of separation of variables to find the solution of the following boundary value problem **(10)**

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x \leq a, \quad 0 \leq y \leq b$$

$$u_x(0, y) = 0, \quad u_x(a, y) = 0,$$

$$u_y(x, b) = 0, \quad u(x, 0) = f(x).$$
